TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

No. 49

ON THE RESISTANCE OF THE AIR AT HIGH SPEEDS AND ON THE AUTOMATIC ROTATION OF PROJECTILES.

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Translated from the French,
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Paris Office, N.A.C.A.

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In the present article the laws governing the flow of a compressible fluid through an opening in a thin wall are applied to the resistance of the air at high speeds.

Let us first consider the reaction, R, of a gaseous jet escaping from a rocket formed of a tube, one end of which is stopped up. At the moment considered, we shall call the pressures and densities inside and outside the tube respectively, p, pa, and ρ, ρa, and we shall take S_1 as the area of the normal section of the tube. Applying the theorem of momentum, we may write

(1) $$R = (p - p_a)\ S_1 = QvS_o$$

in which Q expresses the mass which in a unit of time traverses the unit of area of the true contracted section S_o (sum of the minimum sections of the elementary filaments) of the jet

* Bulletin de L'Institut Aerodynamique de Koutchino, Vol. VI. 1920.

in the proximity of the opening, and v the rate of flow calculated by assuming that the transformation is adiabatic, according to the formula of Saint-Venant

$$(2) \qquad v^2 = \frac{2\,c^2}{\gamma - 1}\left[\left(\frac{p}{p_a}\right)^{\frac{\gamma-1}{\gamma}} - 1\right],$$

in which c is the velocity of sound for the outside pressure and density.

The output Q is equal to $(\rho_a v)$ or to $(\rho_o c_o)$ according as $\frac{v}{c} \lessgtr 1$, the density ρ_o and the speed c_o being connected with the density ρ and the pressure p, inside the tube, by the well known relations

$$(3) \qquad \rho_o = \left(\frac{2}{\gamma + 1}\right)^{\frac{1}{\gamma - 1}} \rho,$$

$$(4) \qquad c_o = \left(\frac{\gamma p_o}{\rho_o}\right)^{1/2} = \left(\frac{2\gamma}{\gamma + 1}\,\frac{p}{\rho}\right)^{1/2}$$

If we take into consideration the equalities (2), (3), and (4), formula (1) may be represented as follows:

$$(5) \qquad \frac{R}{S_1 Q v} = \frac{p - p_a}{Q v} = \frac{S_o}{S_1} = f\left(\frac{v}{c}\right),$$

or

$$(6) \qquad f\left(\frac{v}{c}\right)_{\frac{v}{c} < 1} = \frac{1}{\gamma}\,\frac{c^2}{v^2}\left[\left(\frac{\gamma - 1}{2}\,\frac{v^2}{c^2} + 1\right)^{\frac{\gamma}{\gamma - 1}} - 1\right]$$

and

$$(7) \qquad f\left(\frac{v}{c}\right)_{\frac{v}{c} > 1} = \left(\frac{\gamma^2 - 1}{4\gamma^2}\right)^{1/2}\left(\frac{\gamma + 1}{2}\right)^{\frac{1}{\gamma - 1}}\left(1 + \frac{2}{\gamma - 1}\,\frac{c^2}{v^2}\right)^{1/2}$$

$$\times\left[1 - \left(\frac{\gamma - 1}{2}\,\frac{v^2}{c^2} + 1\right)^{\frac{\gamma}{1 - \gamma}}\right]$$

By introducing the sections S and S' defined by the equalities

$$\frac{S}{S_1} = \frac{S'}{S_o} = 1; \qquad \frac{v}{c} < 1,$$

(8)
$$\frac{S}{S_1} = \frac{S'}{S_o} = \frac{\rho_o c_o}{\rho_a v}; \qquad \frac{v}{c} > 1,$$

we may also represent formula (5) as follows:

(9)
$$\frac{R}{S\rho_a v^2} = \frac{p-p_a}{Qv} = \frac{S'}{S} = f \left(\frac{v}{c}\right)$$

The meaning of this last substitution is as follows: When $v/c < 1$, the pressure in the contracted section S_o is equal to the outside pressure p_a, but when $v/c > 1$, it is equal to

$$p_o = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma-1}} p$$

and the jet must expand still further for the pressure to fall to p_a. As appears from formula (8), S' = the area of the actual section corresponding to such expansion, and S = the area of the section of an imaginary rocket, defined by the equality

$$\frac{S'}{S} = \frac{S_o}{S_1}$$

We thus set down the reaction R, for speeds v either less or greater than that of the sound c, at an outer density of ρ_a.

Assuming in formula (6), v = o, we obtain f(o) = 1/2.

If the fluid were incompressible we should have to assume in formulas (6) and (7) $c = \infty$ and, therefore, in that case, $f(v/c) = 1/2$ for any speed v.

We can represent the reaction R as the sum of two members

$$R = R_1 + R_2$$

in which the first

$$R_1 = \frac{1}{2} S\rho_a v^2$$

depends only on the inertia of the fluid, and the second

$$R_2 = \left[f \left(\frac{v}{c}\right) - \frac{1}{2} \right] S\rho_a v^2$$

expresses the increase in pressure due to the compressibility of the fluid.

Let us try to utilize these results for expressing the resistance R of a body of median section S moving in a compressible fluid at a speed v. In a perfectly incompressible fluid the resistance due to inertia alone is null, for as the energy of the body cannot be transmitted to the medium, the fluid filaments will rejoin each other behind the body and the pressures in the front and rear parts will counter-balance each other. In a perfectly compressible fluid the resistance R will not usually be null, for the energy of the body can be transmitted to the surrounding fluid in the form of waves. The fluid filaments, deviated by the front part of the body, will again join behind it, but the energy of the fluid particles will be lessened in proportion to the number absorbed by the waves. In the extreme case, those parts of the pressure on the front and rear surfaces of the body which are solely due to the inertia of the fluid, will alone counterbalance each other.

If we consider the resistance R of fluids as a phenomenon inverse to that of the reaction studied above, substituting the median section of the body for section S (see formula 9), and taking into account the remark just made, we may admit that

$$\frac{R}{S\rho_a v^2} = f\left(\frac{v}{c}\right) - 1/2$$

This formula can only be true if the waves, due to the elasticity of the fluid, are really formed. Now, when $v/c > 1$, the formation of these waves must take place, for formula (8) which can also be written as follows

$$S'\rho_a v = S_o \rho_o c_o$$

determines the presence of a shock wave, characterized by a sudden change "of section," density and speed in front of the body.* But when $v/c < 1$, there is nothing to lead us to anticipate their formation, and consequently, in this case

$$\frac{R}{S\rho_a v^2} = 0$$

Finally, if we designate by $F(v/c)$ a discontinuous function, equal to $f(v/c) - 1/2$ when $v/c > 1$, and to zero when

* In endeavoring to lessen the resistance due to the formation of waves, we may study perforated projectiles, taking into account formula (8).

$v/c < 1$, we can express the resistance of a perfectly compressible fluid by the formula

$$\frac{R}{S\rho_a v^2} = F(v/c). \tag{10}$$

This curve has a maximum at the point $v/c = 1.74$, which we determine as the root of the equation

$$\left[\frac{\gamma - 1}{2}\left(\frac{v}{c}\right)^2 \qquad 1\right]^{\frac{\gamma}{\gamma-1}} - \gamma\left(\frac{v}{c}\right)^2 - 1 = 0$$

by taking $\gamma = 1.41$. For very high speeds, by taking in formula (10) $v/c = \infty$, we obtain

$$\frac{R}{S\rho_a v^2} = \left(\frac{\gamma^2 - 1}{4\gamma^2}\right)^{1/2}\left(\frac{\gamma + 1}{2}\right)^{\frac{\gamma-1}{1}} - 1/2 = 0.0555$$

In true fluids we must take into account the friction on the walls of the body and also the fact that the relative speed near the walls may be greater than the speed of the projectile and that roughnesses on the walls may then act as small, independent obstables. In this case we may generalize formula (10), say, as follows:

$$\frac{R}{S\rho_a v^2} = a_0 + a_1 F\left(\frac{v}{\theta_1 c}\right) + a_2 F\left(\frac{v}{\theta_2 c}\right) + \ldots\ldots$$

The coefficients $a_0, a_1, a_2, \ldots\ldots$ and $\theta_1, \theta_2, \ldots\ldots$ depend only on the form of the projectile and in terms in which the kinematic viscosity explicitly figures.

As a first approximation, we may take

$$(11) \qquad \frac{R}{S\rho_a v^2} = k + F(v/c),$$

in which k is the coefficient of resistance of the projectile at low speeds. The heavy line discontinuous curve shown in the plate B.1 has been calculated by assuming in formula (11), $k = 0.114$. This is the coefficient given by General N. Maievski in his Ballistice. The dotted curve has been calculated by the formula

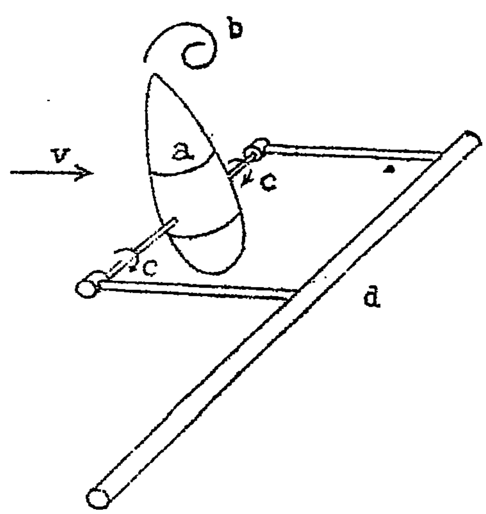
$$\frac{R}{S\rho_a v^2} = k + f(v/c) - 1/2$$

by giving to v/c values < 1 (see formula 6). The points shown on the figure express well known results of experiments made in various countries.

On the Automatic Rotation of Projectiles.

The instability which we observe in projectiles shot into the air without being given a movement of rotation about their axis of symmetry, or without stabilizing planes, is a phenomenon of automatic rotation.

Take a light model of a projectile, a; (see Fig. 1 given below; through the center of gravity, assumed as being not too



near the extremities, pass an axis normal to the axis of symmetry of the projectile and able to turn freely in the centers, cc, of the fork d. Placing the whole device in an air current, as indicated by the arrow, v, we observe that the model may be put in a state of automatic rotation in either direction, according to the initial impulse imparted to it.

We can prevent the occurrence of this phenomena of automatic rotation by bringing the center of gravity sufficiently near one end, or by fitting the projectile with stabilizing planes or a tail. The flight of such projectiles, as fired from trench guns, is, as we know, much more steady.

The automatic rotation of projectiles is due, as I showed in my article on the automatic rotation of rectangular plates,* to the suction produced by the systematic formation of vortices b behind the extremity of the projectile moving with the wind (see figure).

The suction effect of vortices in their incipient stage is well brought out by the experiment on pendular motions in a fluid stream.**

Mr. von Karman's Theory of the Resistance of Fluids*** is also connected with this phenomenon.

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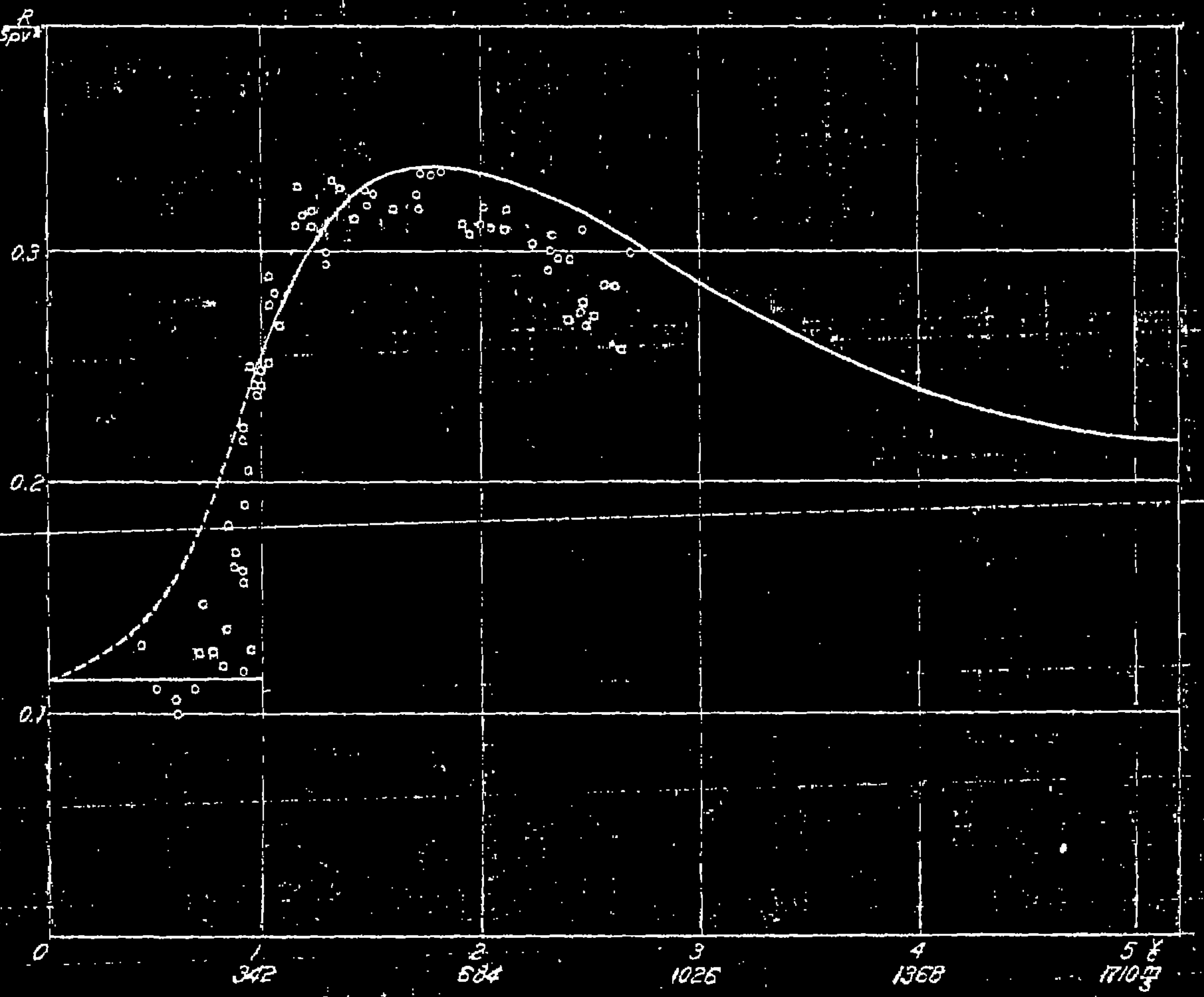
* Bulletin of the Koutchino Aerodynamic Institute, No.II, 1909, p.44, and No.III, 1914, p.89.

** L'Aerophile, January 1, 1911; Bulletin of the Koutchino Aerodynamic Institute, No.IV, 1912, p.91.

*** Göttinger Nachrichten, math.phys.Kl. 1911, p.509; 1912, p.547. Phys. Zeitscher. Vol.XIII 1912, p.49.



Resistance of air at high speeds. Paper by Mr. D. Riabouchinski.